The Opacity of Real-time Automata

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November 1, 2018

Overview

Opacity

Opacity is an information flow property aiming at keeping the secret of a system *opaque* to the intruder with partial observability over its behaviours.

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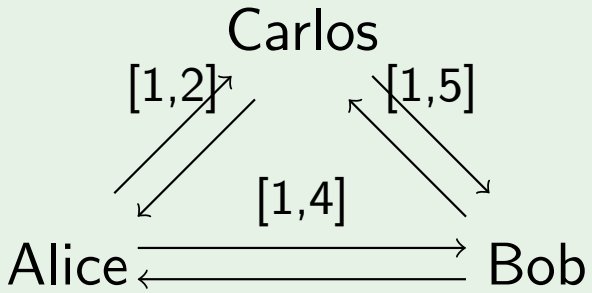

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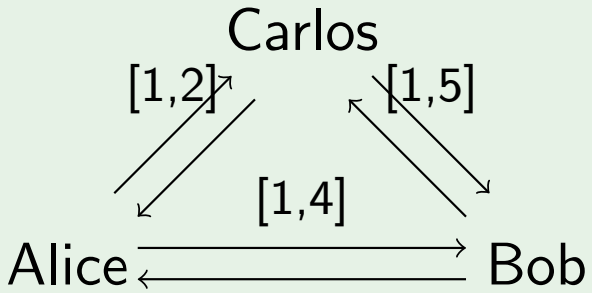

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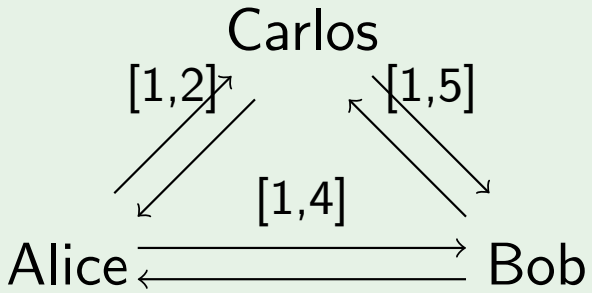

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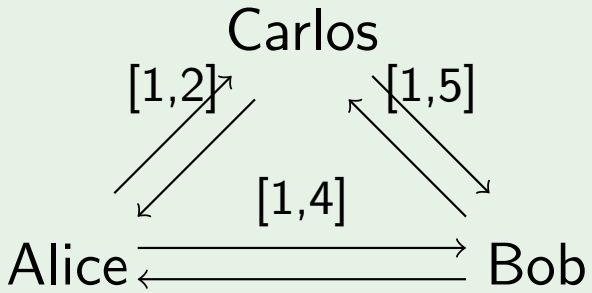

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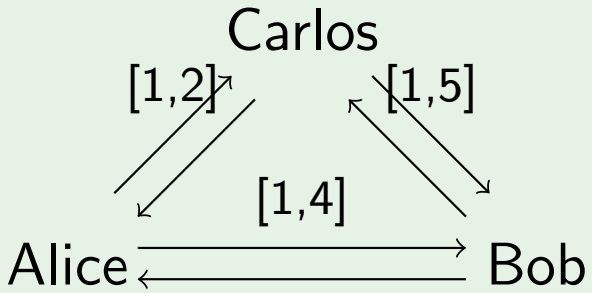

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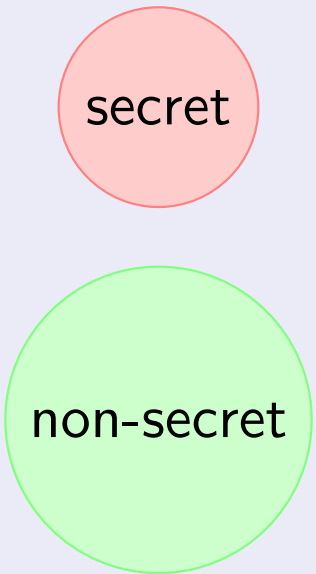

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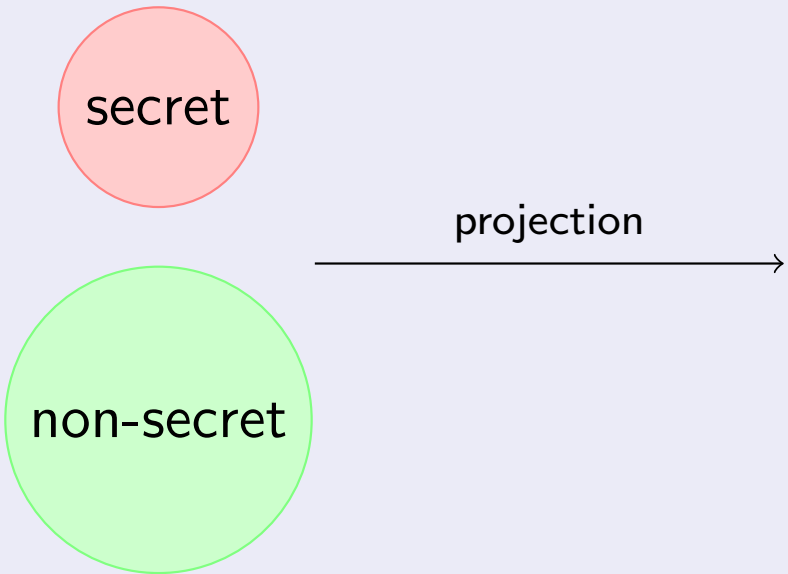
System's behaviours: What the intruder can know:



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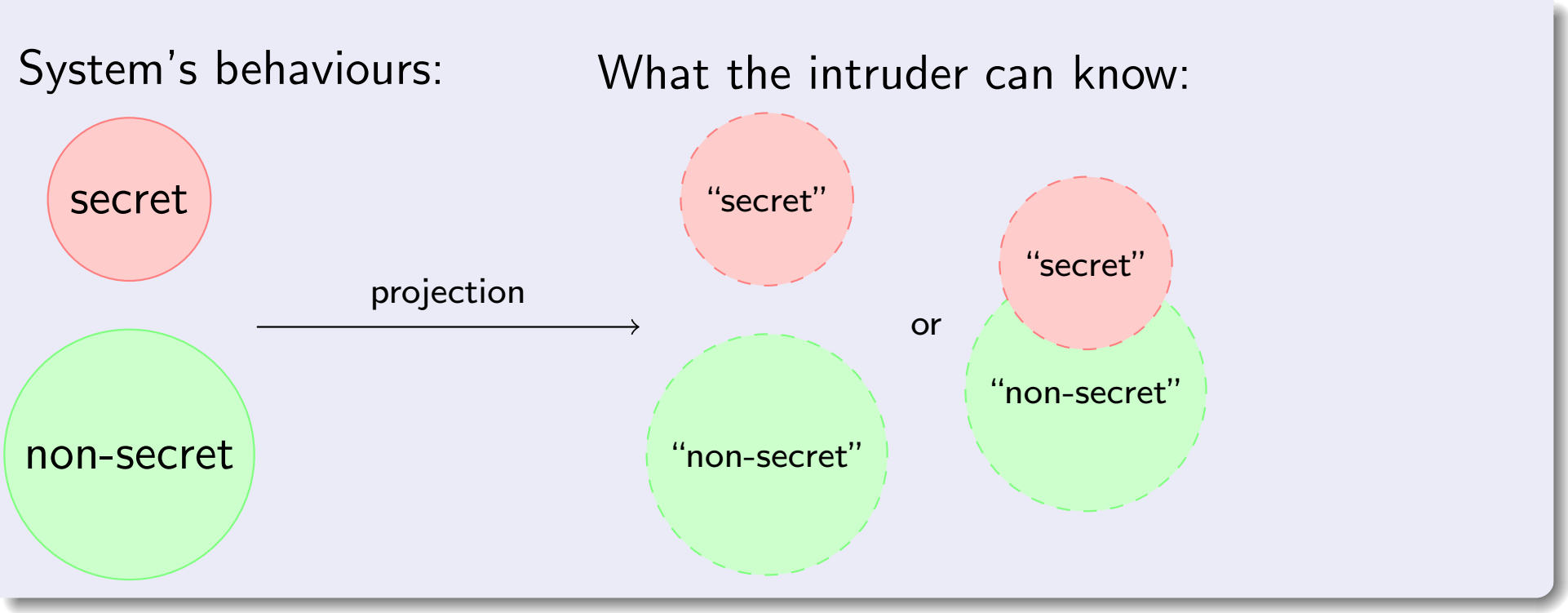

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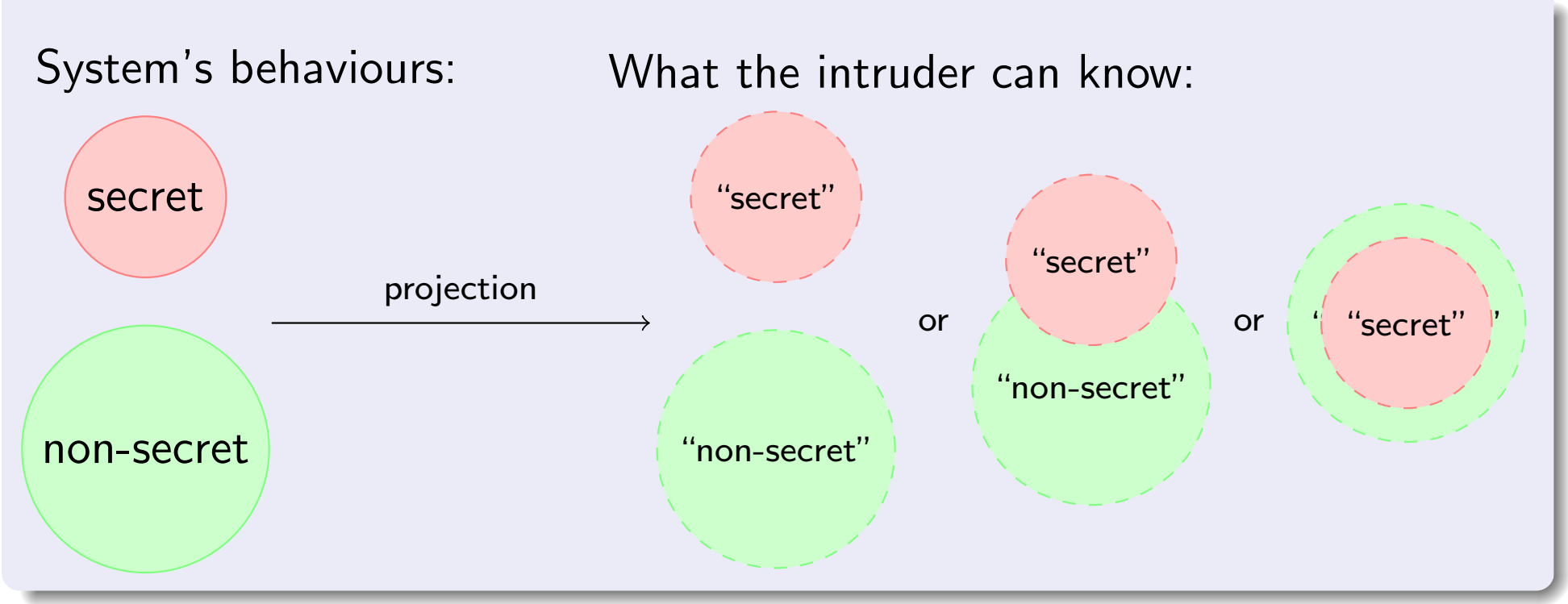

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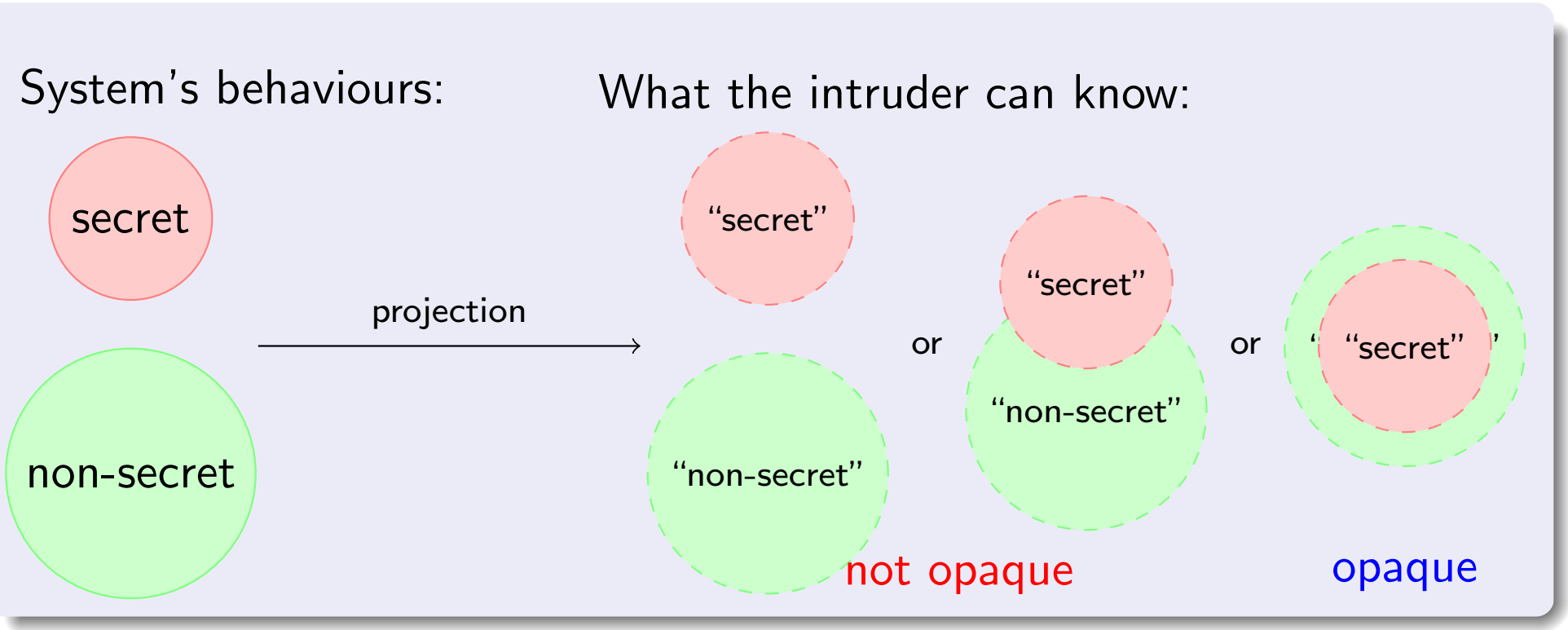

Real-time Automata (RTA)

Definition (Real-time Automata (RTA))

An RTA is a six-tuple $\mathcal{A} = (S, \Sigma, \Delta, \mathit{Init}, F, \mu)$ where

- S is a finite set of states; $\mathit{Init} \subset S$ is initial states; $F \subset S$ is accepting states;
- Σ is a finite alphabet;
- $\Delta \subset S \times \Sigma \times S$ is the transition relation;
- $\mu : \Delta \to 2^{\mathbb{R}_{\geq 0}} \setminus \{\emptyset\}$ is the time labeling function.

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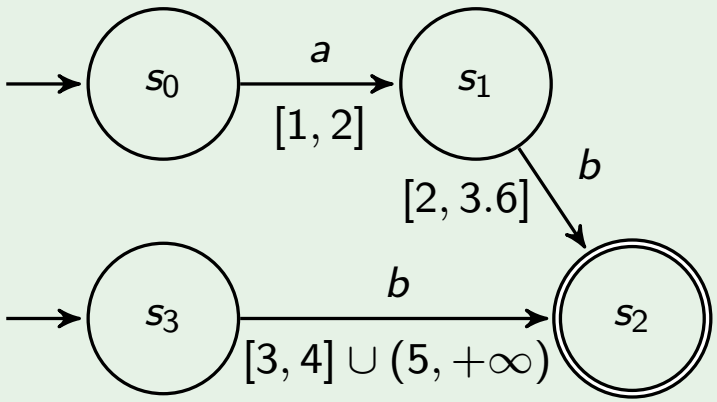
Example (RTA)

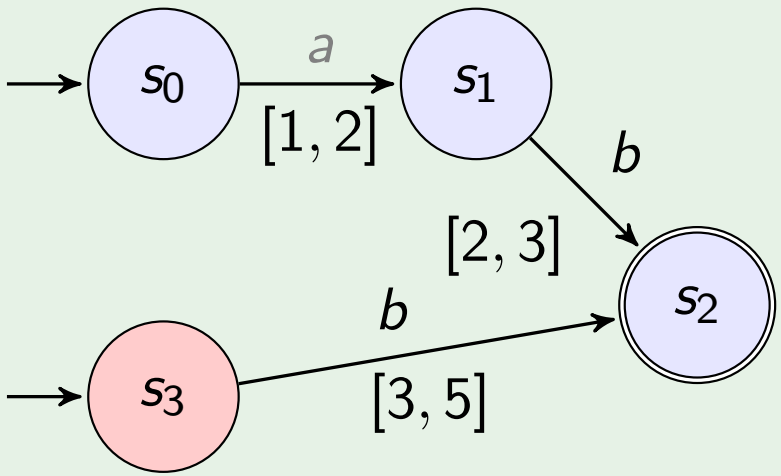


Initial-state Opacity & Language-Opacity of RTA

- **Initial-state Opacity Problem**: The secret is a *subset of its initial-states* of the RTA. Can the intruder never judge whether the run starts from a secret initial-state?

Example

- Let $\{b\}$ be the observable set and s_3 be the "secret" initial state.



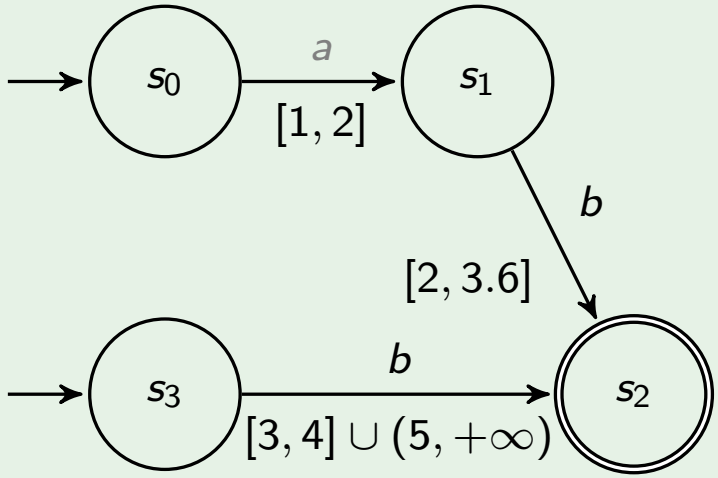
- If (b,t) is observed, possible runs include not only $\rho_0 = s_3 \xrightarrow[t]{b} s_2$, but also $\rho_1 = s_0 \xrightarrow[1]{a} s_1 \xrightarrow[t-1]{b} s_2$, $\cdots$, $\rho_2 = s_0 \xrightarrow[2]{a} s_1 \xrightarrow[t-2]{b} s_2$. So the RTA is initial-state opaque with respect to $\{s_3\}$ and $\{b\}$ in this example.

Initial-state Opacity & Language-Opacity of RTA

- **Language-Opacity Problem**: The secret is a *set of timed words*. Can the intruder never judge whether the run is in the secret set?

Example

The secret is $L_{secret} = \{(a, t_a) \cdot (b, t_b) \mid 1 \leq t_a \leq 3 \wedge 2 \leq t_b \leq 5 \wedge t_a \leq t_b\}$. Again a is unobservable and b is observable.



If $(b, 5)$ is observed, all possible runs are of the form $\rho = s_0 \xrightarrow[t_a]{a} s_1 \xrightarrow[5-t_a]{b} s_2$, where $1 \leq t_a \leq 2 \wedge 2 \leq 5 - t_a \leq 3.6$, i.e., the corresponding traces are $\{(a, t_a) \cdot (b, 5) \mid 1 \leq t_a \leq 1.4\} \subset L_{secret}$.

So the RTA is not language-opaque in this case.

Initial-state Opacity & Language-Opacity of RTA

Initial-state opacity of RTA

$\mathcal{A}$ is initial-state opaque with respect to S_{secret} and Σ_o iff $P_{\Sigma_o,t}(Traces\,(Init \cap S_{secret}))) \subseteq P_{\Sigma_o,t}(Traces\,(Init \setminus S_{secret}))$, equivalently, iff $P_{\Sigma_o,t}(L(\mathcal{A})) \subseteq P_{\Sigma_o,t}(Traces\,(Init \setminus S_{secret}))$.

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Language-opacity of RTA

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Theorem (1)

An RTA $\mathcal{A}$ is initial-state opaque with respect to S_{secret} and Σ_o iff it's language-opaque with respect to $L(\mathcal{A}) \setminus Traces\,(Init \setminus S_{secret})$ and Σ_o

Deciding the opacity of RTA

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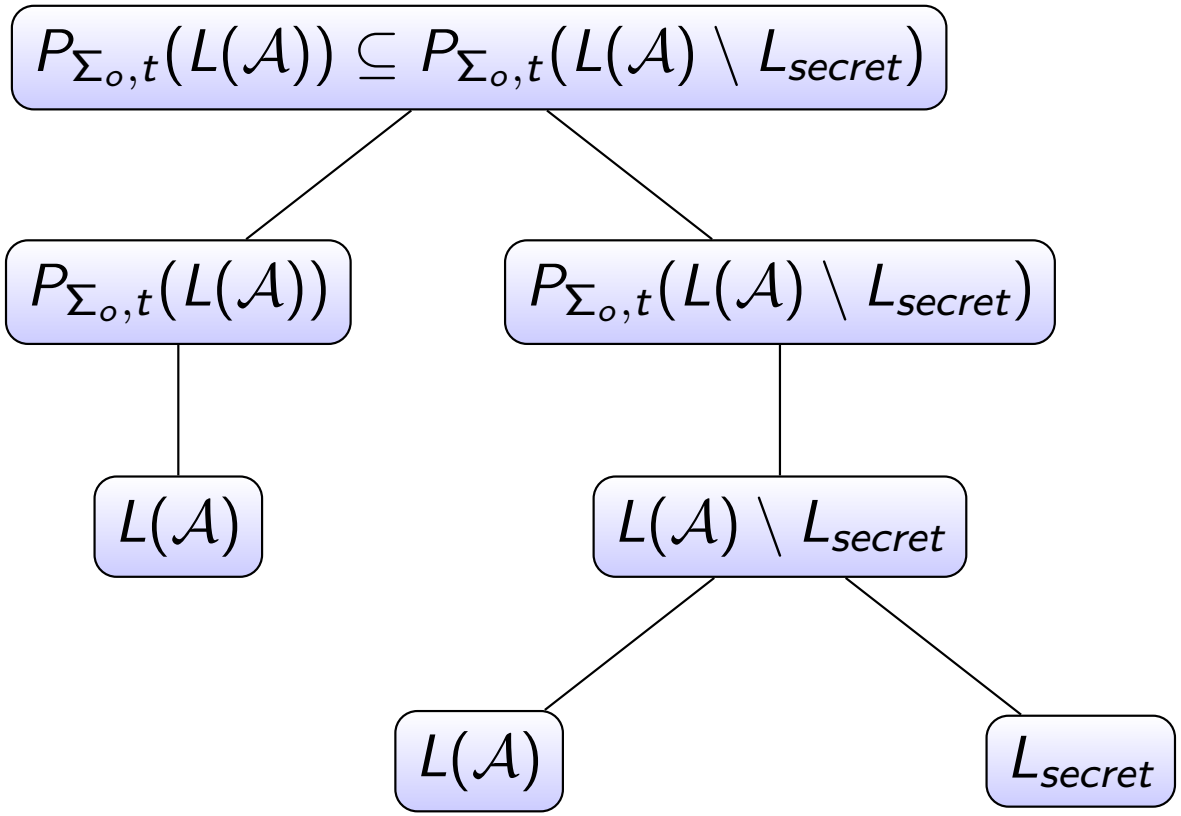
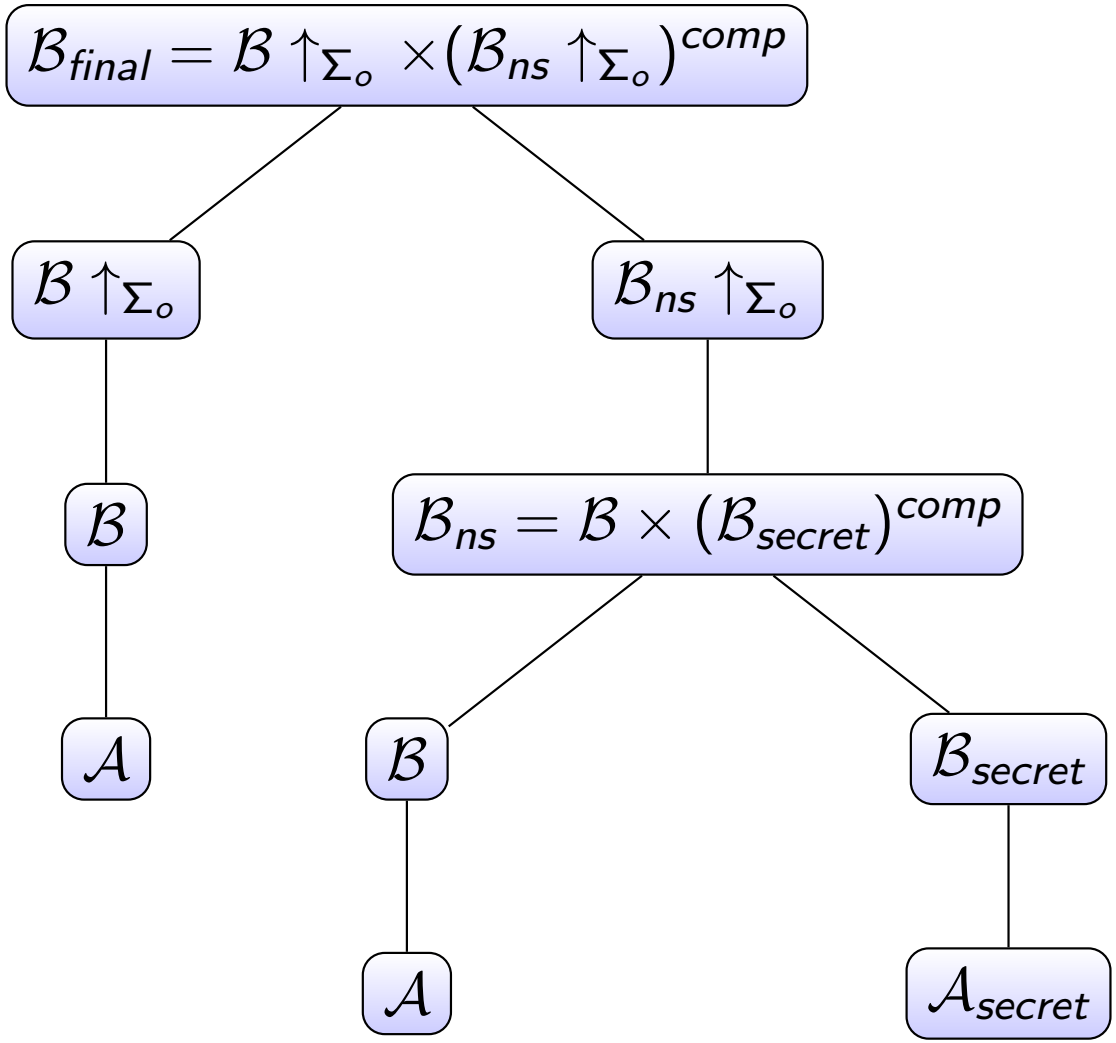


The Whole Progress



Main methods

Translating RTA into FA

- the relation of trace-equivalence
- the partitioned alphabet and partitioned language

Computing projection of FA

- summation of successive unobservable durations
- merging unobservable durations into observable transitions' time labels

The Relation of Trace-Equivalence — $L_{FA} \approx_{tr} L_{RTA}$

- The languages of the newly-obtained FA $\mathcal{B}$ and of the original RTA $\mathcal{A}$ should represent each other.

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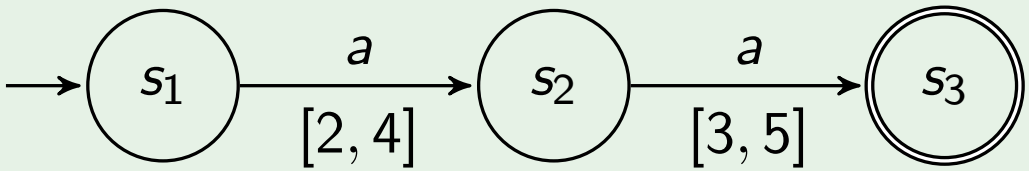
Example

$L_f(\mathcal{A}) = \{(a, t_1) \cdot (a, t_2) \mid t_1 \in [2, 4] \wedge t_2 - t_1 \in [3, 5]\}$



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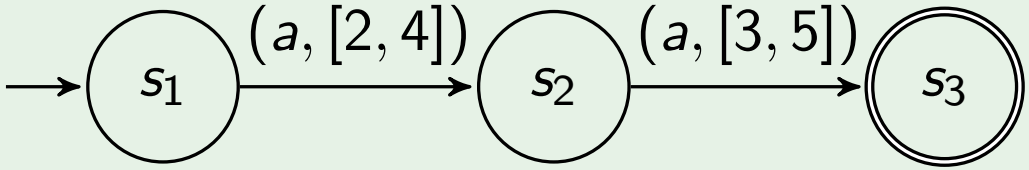

$L_f(\mathcal{B}) \approx_{tr} L_f(\mathcal{A})$

- However, the relation of trace-equivalence is not preserved when it comes to the complement and product operations of FA.

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Example (Ctd.)

The complement of $L_f(\mathcal{A})$ is

$$\{\varepsilon\} \cup \{(a,t) \mid t \in [0,+\infty)\} \cup$$
$$\{(a,t_1)\cdot(a,t_2) \mid t_1 \notin [2,4] \vee t_2 - t_1 \notin [3,5]\} \cup \ldots$$

while the complement of $L_f(\mathcal{B})$ is

$$\{\varepsilon\} \cup \{(a,[2,4]),(a,[3,5])\} \cup$$
$$\{(a,[2,4])\cdot(a,[2,4]),(a,[3,5])\cdot(a,[3,5]),(a,[3,5])\cdot(a,[2,4])\} \cup \ldots$$

Not trace-equivalent at all!

Put restrictions on the timed alphabet

Therefore, some restrictions should be put on the timed alphabet:

Partitioned alphabets and partitioned languages

Let the timed alphabet be $\Sigma_t = \bigcup_{\sigma \in \Sigma} \{\sigma\} \times T_\sigma$.
An alphabeta Σ_t is called a **partitioned alphabet** if for each σ, T_σ is a partition of $\mathbb{R}_{\geq 0}$.
And language over partitioned alphabets are called **partitioned langauges**.

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Lemma (Trace-equivalence preserved under complement)

$(\Sigma_t^* \setminus L_2) \approx_{tr} (TW^*(\Sigma) \setminus L_1)$ if L_1 is a timed language over Σ, L_2 is a partitioned language over Σ_t with $L_2 \approx_{tr} L_1$.

Lemma (Trace-equivalence preserved under intersection)

$(L_{21} \cap L_{22}) \approx_{tr} (L_{11} \cap L_{12})$ if L_{11} and L_{12} are timed languages over Σ, L_{21} and L_{22} are partitioned languages over Σ_t, $L_{21} \approx_{tr} L_{11}$ and $L_{22} \approx_{tr} L_{12}$.

Put restrictions on the timed alphabet

Example

Let $\Sigma_t = \{(a,[2,3)),(a,[3,4]),(a,(4,5]),(a,[0,2)\cup(5,+\inf))\}$.
(Σ_t is partitioned.)

$$L_{21} = \{(a,[3,4]),(a,(4,5])\} \approx_{tr} \{(a,t) \mid t \in [3,5]\} = L_{11}$$
$$L_{22} = \{(a,[2,3)),(a,[3,4])\} \approx_{tr} \{(a,t) \mid t \in [2,4]\} = L_{12}$$
$$L_{21} \cap L_{22} = \{(a,[3,4])\} \approx_{tr} \{(a,t) \mid t \in [3,4]\} = L_{11} \cap L_{12}$$
$$L_{21}^c \approx_{tr} L_{11}^c$$

Summary: Translating RTA into FA

1. Moving time labels into actions to obtain a new FA;
2. Splitting transitions such that the alphabet of the FA is partitioned.

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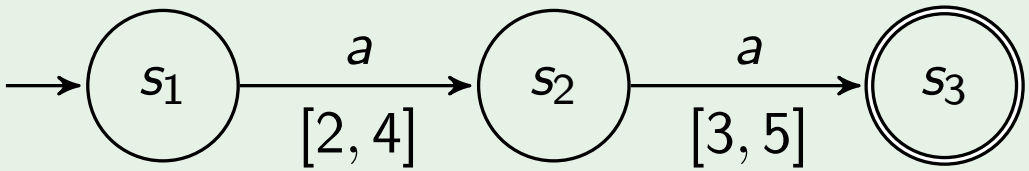

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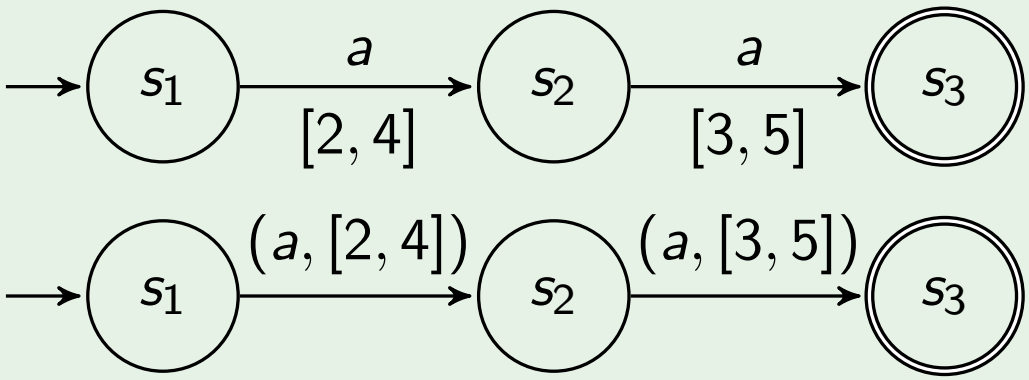

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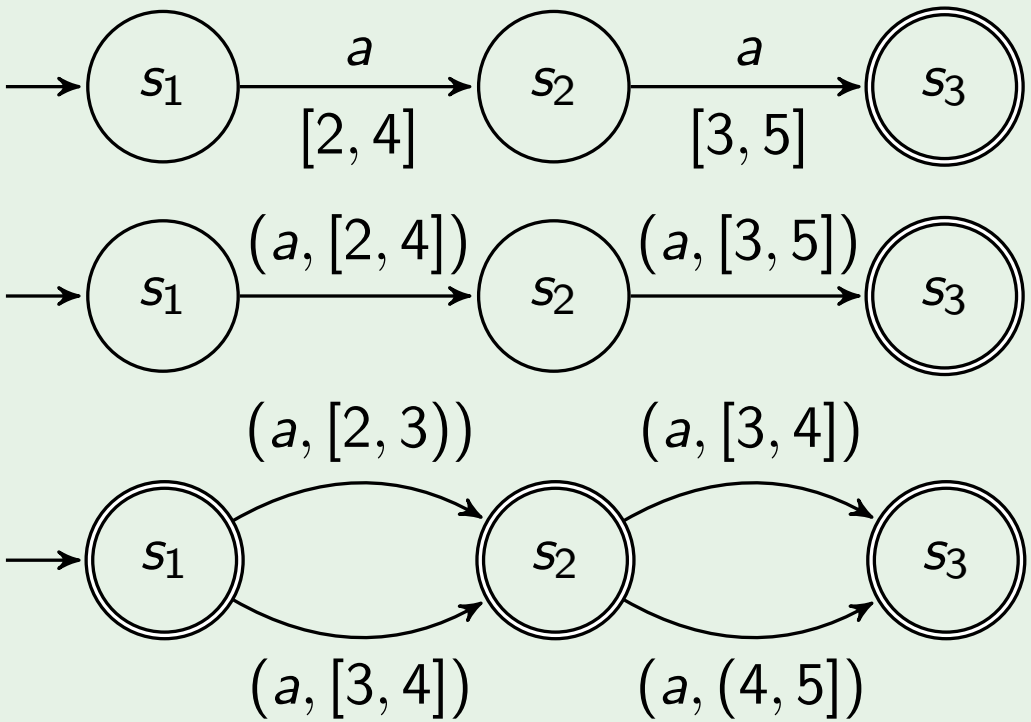


Projection of words and languages

Definition

$P_{\uparrow\Sigma_o}(w)$ is inductively defined based on the number of observable symbol-duration pairs in w:

$$P_{\uparrow\Sigma_o}(u^1) = \varepsilon;$$

$$P_{\uparrow\Sigma_o}(u^1 \cdot (a_1, \Lambda_1)) = \begin{cases} (a_1, \Lambda_1), & \text{if } u^1 = \varepsilon \\ (a_1, \sum_{k=1}^{m_1} \Lambda_{1k} + \Lambda_1), & \text{otherwise} \end{cases}$$

$$P_{\uparrow\Sigma_o}((u^1 \cdot (a_1, \Lambda_1)) \cdot w_{suffix}) = P_{\uparrow\Sigma_o}(u^1 \cdot (a_1, \Lambda_1)) \cdot P_{\uparrow\Sigma_o}(w_{suffix})$$

where u^1 is either ε or $(\tau, \Lambda_{11}) \ldots (\tau, \Lambda_{1m_1})$.
And $P_{\uparrow\Sigma_o}(L) = \{P_{\uparrow\Sigma_o}(w) \mid w \in L\}$.

Lemma (Trace-equivalence preserved under projection)

If $L_2 \approx_{tr} L_1$, then $P_{\uparrow\Sigma_o}(L_2) \approx_{tr} P_{\Sigma_o,t}(L_1)$.

Projection of words and languages

Example

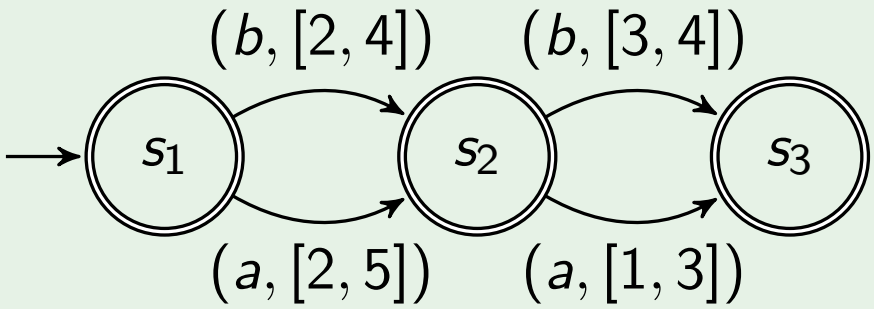
- τ represents all unobservable actions and $a_1, \cdots, a_n$ are observable.

$$
\begin{aligned}
w_t =& (\tau, 0.2)(\tau, 0.5)(\tau, 0.9) \cdot (a_1, 1) \cdot \\
& (\tau, 1.2)(\tau, 1.7) \cdot (a_2, 2) \ldots \\
& (\tau, n-0.5)(\tau, n-0.2) \cdot (a_n, n) \cdot \\
& (\tau, n+0.2) \\
P_{\Sigma_o, t}(w_t) =& (a_1, 1) \cdot (a_2, 2) \cdot \ldots \cdot (a_n, n) \\
w =& (\tau, [0, 1, 0.3])(\tau, [0, 0.4])(\tau, [0.2, 0.4]) \cdot (a_1, [0, 0.2]) \cdot \\
& (\tau, [0.1, 0.3])(\tau, [0.3.0.6]) \cdot (a_2, [0.2, 0.4]) \cdots \\
& (\tau, [0.3, 0.6])(\tau, [0.3, 0.6]) \cdot (a_n, [0, 0.2]) \cdot \\
& (\tau, [0.2, 0.4]) \\
P_{\uparrow \Sigma_o}(w) =& (a_1, [0.3, 1.3]) \cdot (a_2, [0.6, 1.3]) \cdots (a_n, [0.6, 1.4])
\end{aligned}
$$

Summing up Successive Unobservable Durations

Example

Let $\Sigma_o = \{a\}$.



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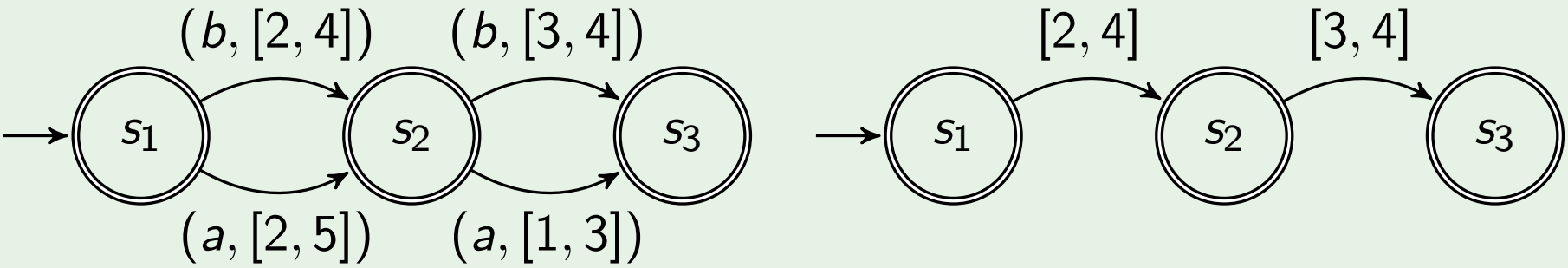

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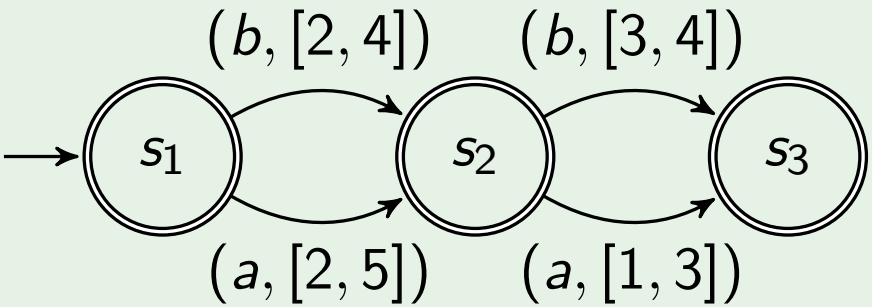
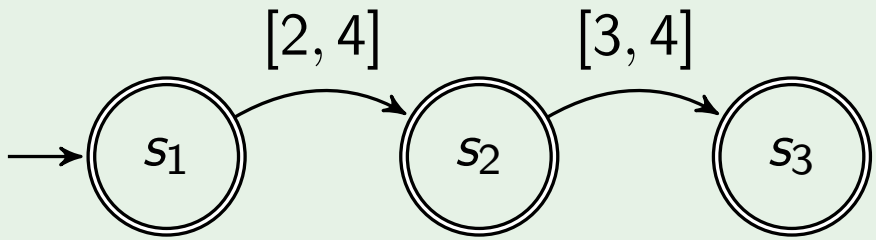




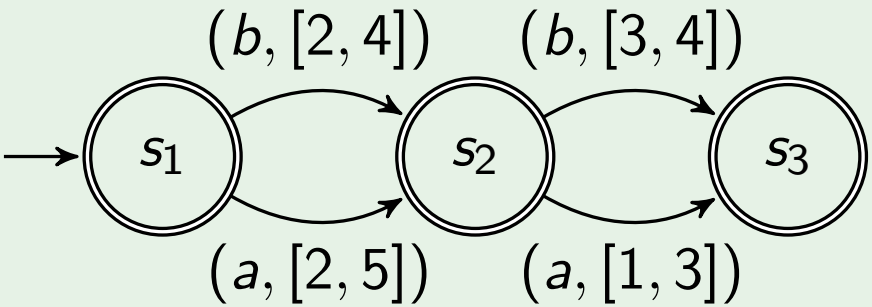
Regular expressions

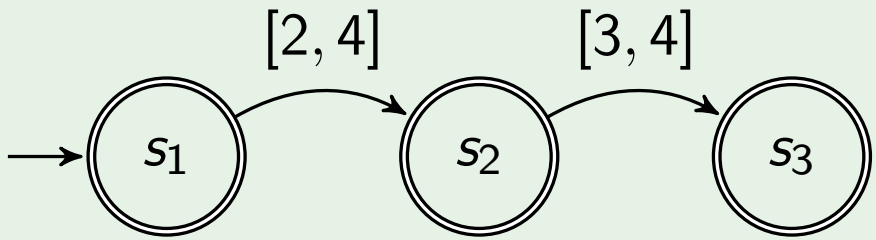
	s_1	s_2	s_3
s_1	ε	$[2,4]$	$[2.4] \cdot [3,4]$
s_2	$\emptyset$	ε	$[3,4]$
s_3	$\emptyset$	$\emptyset$	ε

Summing up Successive Unobservable Durations

Example

Let $\Sigma_o = \{a\}$.





Regular expressions

	s_1	s_2	s_3
s_1	ε	$[2, 4]$	$[2.4] \cdot [3, 4]$
s_2	$\emptyset$	ε	$[3, 4]$
s_3	$\emptyset$	$\emptyset$	ε

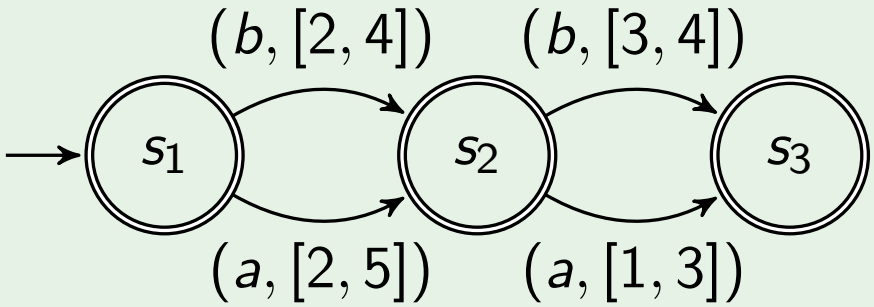
Durations

	s_1	s_2	s_3
s_1	$[0, 0]$	$[2, 4]$	$[6, 8]$
s_2	$\emptyset$	$[0, 0]$	$[3, 4]$
s_3	$\emptyset$	$\emptyset$	$[0, 0]$

Merging Unobservable Durations into Observable Transitions

Example

According to the original FA and the durations calculated before,

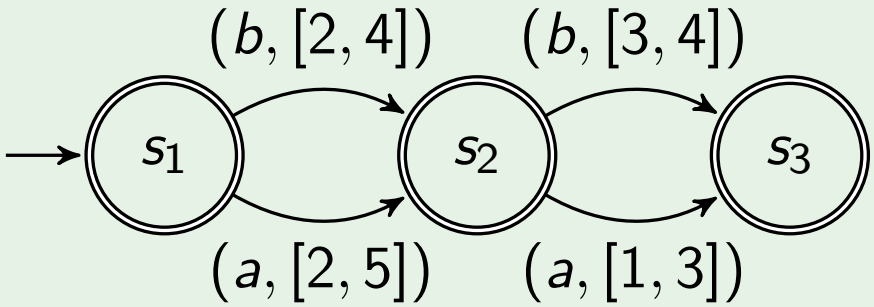


	s_1	s_2	s_3
s_1	$[0, 0]$	$[2, 4]$	$[6, 8]$
s_2	$\emptyset$	$[0, 0]$	$[3, 4]$
s_3	$\emptyset$	$\emptyset$	$[0, 0]$

Merging Unobservable Durations into Observable Transitions

Example

According to the original FA and the durations calculated before,



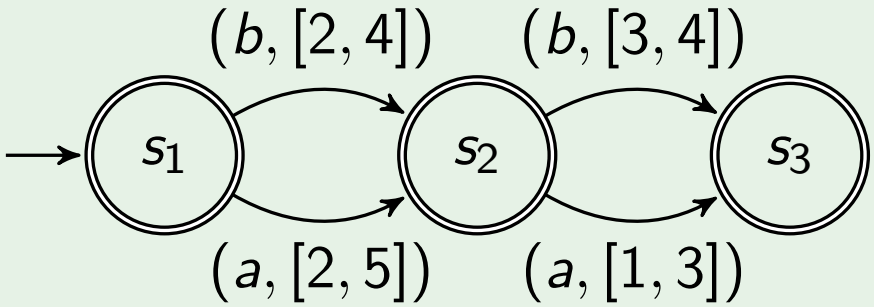
	s_1	s_2	s_3
s_1	$[0, 0]$	$[2, 4]$	$[6, 8]$
s_2	$\emptyset$	$[0, 0]$	$[3, 4]$
s_3	$\emptyset$	$\emptyset$	$[0, 0]$

the projection FA is as follows:

Merging Unobservable Durations into Observable Transitions

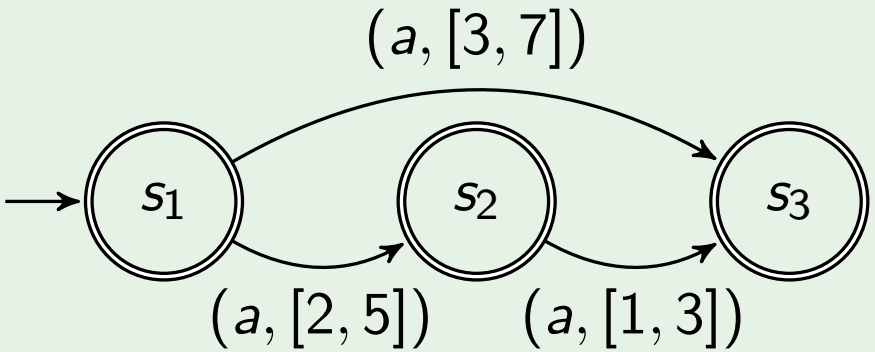
Example

According to the original FA and the durations calculated before,



	s_1	s_2	s_3
s_1	$[0, 0]$	$[2, 4]$	$[6, 8]$
s_2	$\emptyset$	$[0, 0]$	$[3, 4]$
s_3	$\emptyset$	$\emptyset$	$[0, 0]$

the projection FA is as follows:

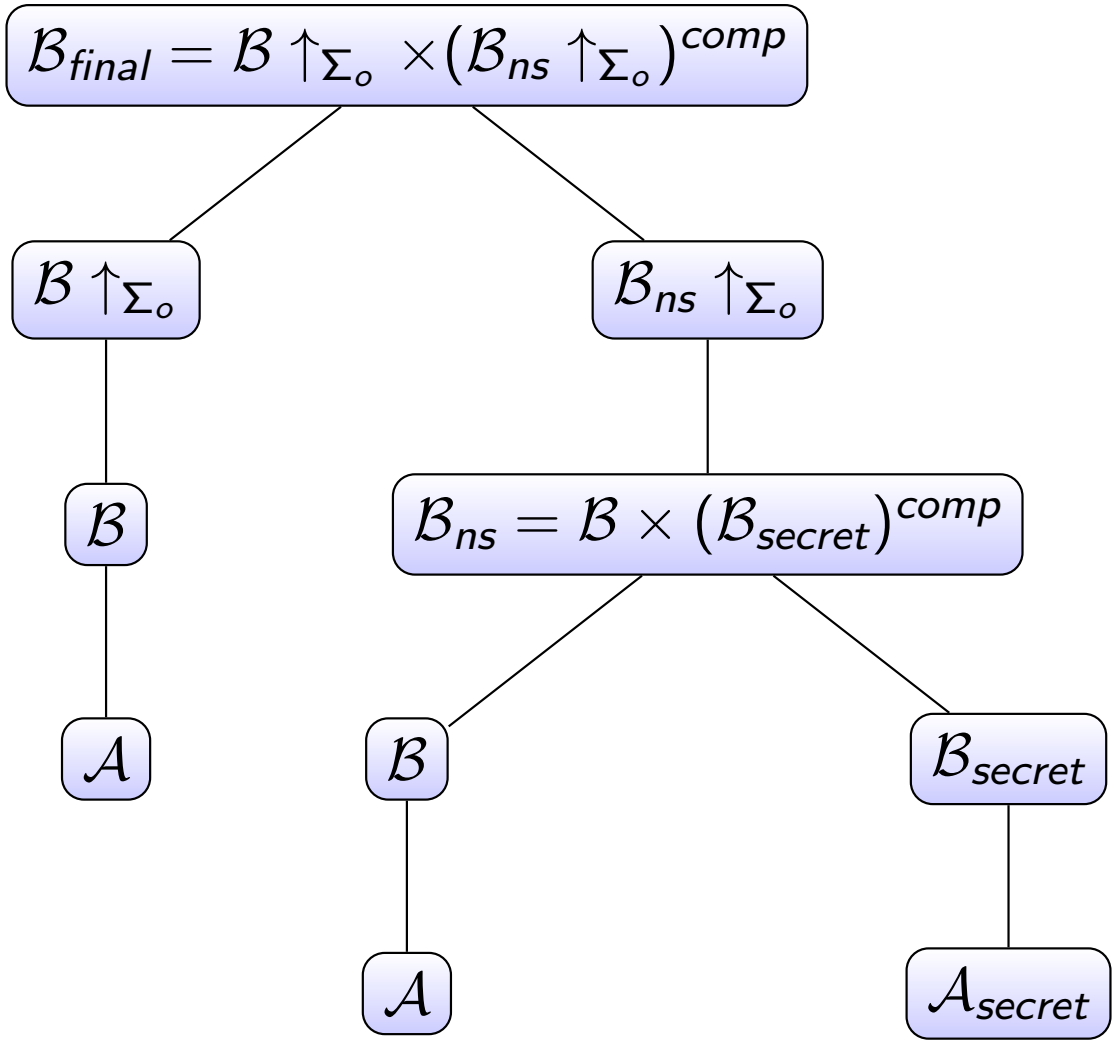


Summary: Computing Projection of FA

1. Summing up successive unobservable durations (using Dima's Normal Form[a] of intervals of Int)
2. Merging unobservable durations into observable transitions' time labels

[a] Dima, C: *Real-time automata.* Journal of Automata, Languages and Combinatorics 6(1), 3-24 (2001)

The Whole Progress



Decidability of Language-Opacity and Initial-State Opacity of RTA

Theorem (2)

The language-opacity problem of RTA is decidable.

Theorem (3)

The initial-state opacity problem of RTA is decidable.

The End

Thank you!